# Y2 primary mathematics: developing the mathematics curriculum (Terms 1, 2 and 3)

NB: Content will be added as the seminars progress across the year.

This module builds on the mathematical knowledge you developed during year 1 of your studies. In year 2, we are looking at how children's learning typically develops across Key Stage 2. You will be supported and encouraged to develop your conceptual understanding as this will impact your subject knowledge, to include understanding how children's learning typically progresses in maths. We will consider effective pedagogical approaches to support teaching and learning, and ask you to reflect on your teaching practices from your first year of university, using research to support your progression in thinking.

#### 88 items

# How to make the most of this reading list in relation to your assignment (1 items)

The reading list has been divided into sections linked to your lectures/seminars on campus; it is comprehensive and has additional sections at the bottom of the page. Depending on your assignment focus, the materials will also support to engage with relevant materials. It is our expectation, however, that you read around and beyond this list and this reading should be academic rather than related to websites.

Crucially important is that you engage critically with the materials you read - recognising that one journal article or one book is one opinion. Just as there is no one resource that is perfect in maths, there is no one article/book that will fulfil all of your needs. For example, if I were looking at mastery, I might believe that it involves workbooks for the class, particularly if that's what the scheme of work provides. Now, I know that I could easily find written materials to back this up but is it really the best way to teach for mastery or is it just what we're seeing more and more of in schools? My assignment would, therefore, need to explain why worksheets are great for keeping children busy in lessons but not necessarily great for learning (with evidence to back this up from academic sources). Doing this would evidence my criticality.

# Core documents (5 items)

Mathematics explained for primary teachers, by Derek Haylock; Ralph Manning, 2019 Book | Essential | This is a core text for the module and will be a very helpful source of information while on placement. You are advised to buy a copy of this book as it will be



useful throughout the course and will be using it in sessions. No e-book is available.

# Small numbers, big ideas : essential concepts for teaching early maths, by Jonathan Austen, 2024

Book | Recommended

**Understanding mathematics for young children: a guide for teachers of children 3-7**, by Derek Haylock; Anne Cockburn, 2017

**Book** | Recommended | For children to understand and effectively engage with mathematics in KS2, this needs to be underpinned by sound and varied experiences of mathematics within KS1. Understanding the way thinking develops in younger children will support you to see the way progression develops. You are advised to buy a copy of this book as it will be useful throughout the course. No e-book is available as the publisher refuses to sell the e-book version to libraries.

Improving Mathematics in the Early Years and Key Stage 1: Five recommendations to support practitioners in developing the maths skills of 3–7 year-olds, by Education Endowment Foundation, 2020

Webpage | Recommended | Discussed in the session, this guidance focuses schools on what is considered important in terms of teaching maths in EY and KS1 - the use of manipulatives, quality targeted support and the integration of maths across the day. The poster guidance will provide useful assurances when teaching.

Improving Mathematics in Key Stages 2 and 3: Eight recommendations to improve outcomes in maths for 7–14 year olds, by Education Endowment Foundation, 2017 Webpage | Recommended | Discussed in the session, this guidance focuses schools on what is considered important in terms of teaching maths in KS2 (and KS3) maths strategies, connections, structured interventions and, contrary to some, the use of manipulatives and pictorials. The poster guidance will provide useful assurances when teaching.

# Progression documents (4 items)

National curriculum in England: primary curriculum, by Department for Education, 2015 Webpage | Essential | This document outlines the entitlement of any child (i.e. a statutory requirement) - it should become a guiding document for you whenever you're teaching, irrespective of whether your school follows a scheme or has its own curriculum. Download this document in its entirety and use it, rather than specific maths versions, to support your teaching in years 1-6. Working with the document in this way will increase your familiarity with all of it, and support you to make connections between maths and other subjects - the more we can link teaching, the more effective it will be. Even if you are teaching in nursery or EY, please do refer to the year 1 skills - it shows how learning progresses.

Early years foundation stage (EYFS) statutory framework The standards that school and childcare providers must meet for the learning, development and care of children from birth to 5., by Department for Education, 2024 Document | Essential

Guidance on the teaching of mathematics in primary schools, by Department for Education

#### 2020

Webpage | Essential | Released in July 2020, this DfE document introduces the NCETM's work around what it calls the 'Ready to Progress Criteria' (RtPs). The premise behind this is that there are certain non-negotiables in maths that children need to understand, and that children should only be progressed to these when they are ready. Essentially, it involves teaching less to support deeper understanding, which is why some aspects of maths are noticeable in their absence. Navigtating these materials can be tricky at first, in part due to the various abbreviations, but once you have a handle on these, you will find it to be a very helfpul resource for your teaching practice.

Exemplification of ready-to-progress criteria, by NCETM Webpage

Mastery and the role of fluency, reasoning and problem solving  $(\mbox{21}\)$  items)

A Time for Telling - in Cognition and Instruction, by Daniel L. Schwartz, 19980101 Article | Optional | This article reflects on findings that students learn more when encouraged to work intuitively on problems that would typically require a method or algorithm, before such time as they are even taught the methods. This meant that when students did eventually encounter the method, their brains were ready to engage with it. It's particularly relevant given the NC emphasis on formal, efficient methods which are often taught to the detriment of informal and inefficient methods that, in those formative stages, make perfect sense to learners.

Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching, by Jo Boaler; Carol S. Dweck, 2015 Book | Recommended

**Primary Mastery professional development**, by National Centre for Excellence in the Teaching of Mathematics

Webpage | Recommended | The NCETM urges teachers to use the teaching guidance alongside the animated Powerpoint slides, as teaching opportunities will otherwise be missed. These resources are free of charge and highly effective when whole class teaching.

**Relational Understanding and Instrumental Understanding** - in Mathematics Teaching, by Richard R Skemp, 1976

**Article** | **Recommended** | On the 'Papers' section of Skemp's website is his article entitled 'Relational Understanding and Instrumental Understanding' - one of the most cited articles in maths education. The article focuses on the difference between teaching for understanding (relational) and teaching rules (instrumental). This will. hopefully, be one of those articles that forever resonates with you.

#### NCETM primary assessment materials, by NCETM, 2015

**Webpage** | Essential | These materials are designed to support your teaching. Examples of mastery and mastery at greater depth are provided, which are useful tasks (when chosen wisely) to demonstrate learning in lesson/ series of lessons. Questions are ideal for extension tasks in lessons and/or for exit and entrance tickets.

The Mathematics Teacher Exchange and 'Mastery' in England: The Evidence for the Efficacy of Component Practices - in Education Sciences, by Mark Boylan; Bronwen Maxwell; Claire Wolstenholme; Tim Jay; Sean Demack

Article | Recommended | Offering a different perspective on mastery, this document is a useful read.

**Mastery and depth in primary mathematics: enriching children's mathematical thinking**, by Fay Lewis; Amanda Wilkinson; Marcus Witt, 2022

**Book** | **Recommended** | This book is highly recommended to support you to understand what mastery looks like in school. It is easy to read and is very up to date.

#### Subject report series: maths, by Ofsted, 2023

**Webpage** | **Optional** | Published in July 2023, this Ofsted report about maths teaching (primary and secondary observations) will be repeatedly referenced in seminars.

#### Fluency (4 items)

**Procedural Fluency in Mathematics**, by National Council of Teachers of Mathematics, 2014 **Webpage** | Essential | There is so much misunderstanding around procedural fluency and this document clarifies it.

Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts, by Jo Boaler, 2015

**Document** | To be inspired, read this article.

**Developing Number Fluency - What, Why and How**, by Lynne McClure, 2014 **Document** | Recommended | Focusing on efficiency, accuracy and flexiblity, this article succinctly explains why fluency is important and, hopefully, debunks the myth that it is all about speed.

**Combined fluency and cognitive strategies instruction improves mathematics achievement in early elementary school** - in Contemporary Educational Psychology, by Martha Carr; Gita Taasoobshirazi; Rena Stroud; James M. Royer, 2011-10

Article | This research discusses the impact of using a range of strategies.

# Reasoning (3 items)

**Development of maths capabilities and confidence in primary school**, by Terezinha Nunes, 2009

**Document** | Optional | The importance of children's reasoning on later achievement in mathematics

Reasoning: the Journey from Novice to Expert, by NRICH Primary Team, 2021 Document | Optional | Progression in reasoning

Developing opportunities and ensuring progression in the development of reasoning skills, by NCETM

**Webpage | Optional** | Specific reasoning activities linked to strands of maths

# Problem solving (6 items)

**Teaching specific tactics for problem solving** - in Foster77 Mathematics Education: Colin Foster's mathematics education blog, by Colin Foster, 2023-03-30 **Webpage** | **Recommended** | This blogpost by Dr Colin Foster discusses problem solving as it has been explained in sessions.

Habits of Mind: An Organizing Principle for Mathematics Curricula. - in Journal of Mathematical Behavior, by Al Cuoco; Paul Goldenberg; June Mark, 1996

Article | Optional | This article talks about learners being tinkerers, experimenters, pattern sniffers etc. A really worthwhile read if you want to understand the importance of exploring problems, describing challenges, identifying possible solutions and their implications, and making decisions about how to proceed.

#### Primary Teachers, by NRICH

Webpage | Optional | Low threshold, high ceiling tasks (accessible to all and challenging to all)

**Enhancing students' mathematical problem-solving skills through bar model visualisation technique** - in International Electronic Journal of Mathematics Education, by Sharifah Osman; Che Nurul Azieana Che Yang; Mohd Salleh Abu; Norulhuda Ismai; Hanifah Jambari; Jeya Amantha Kumar

Article | Optional | A useful article outlining the impact of bar modelling on problem solving

Understanding and enriching problem solving in primary mathematics, by Patrick Barmby; David Bolden; Lynn Thompson, 2014 Book | Optional

The effects of feedback during exploratory mathematics problem solving: Prior knowledge matters - in Journal of Educational Psychology, by Emily R Fyfe; Bethany Rittle-Johnson; Marci S DeCaro, 2012

Article | Optional | Using feedback effectively when problem solving

# Session 2: Counting (2 items)

Learning and Teaching with Learning Trajectories

Website | Recommended | This website remains free of charge but does require you to sign up - no strings attached. You can access materials by area (e.g. counting) and there are plenty of activities to support teaching, but the real strength of this website is the learning trajectories (the typical stages of development when children are learning maths). These learning trajectories are diagnostic and will help you to recognise the preceding skills involved when teaching a concept which, when plugged, will enable children to progress to the next stage of learning. There are also very practical formative assessment materials available on the website (small group and 1-1 tools) that will support you to see what children have grasped.

**Compressing the counting process: strength from the flexible interpretation of symbols** - in Teaching and Learning Early Number, by Eddie Gray, 2008

**Chapter** | **Optional** | Gray on the counting trap

#### Session 3: Place value (4 items)

Place value: the English disease? - in Enhancing primary mathematics teaching, 2003 Chapter || Recommended

What is unitising, and why is it important?, by National Centre for Excellence in the Teaching of Mathematics, 2019-11-06

Webpage || Recommended

The acquisition of numeracy, by Jenny Young-Loveridge Document ]| Optional

An investigation of the relationship between young children's understanding of the concept of place value and their competence at mental addition, by Ian Thompson; Rod Bramald

Document

#### Session 4: Addition and subtraction (5 items)

Calculation guidance for primary schools, by National Centre for Excellence in the Teaching of Mathematics, 2015

Document || Recommended

Addition and subtraction mental calculation strategies

Webpage | Recommended | This is a link to a really useful poster which details additive mental maths strategies (informal)

Subtraction: Lesson videos on elements of progression in subtraction in Key Stage 2, by NCETM

Webpage || Recommended | These video clips from KS2 show clearly progression from lower KS2 to upper KS2 and the use of representatives to move children to compact methods with understanding.

Primary mathematics for trainee teachers, edited by Marcus Witt, 2014 **Book** | Optional | Chapter 4 provides a good overview of addition, inlcuding the most effective representations.

The effect of instruction on children's solutions of addition and subtraction word problems in Educational Studies in Mathematics : An International Journal, by T. P. Carpenter, 198302

Article || Recommended

#### Session 5: Assignment support (1 items)

The contribution of educational research to teachers' professional learning: philosophical understandings - in Oxford Review of Education, by Christopher Winch; Alis Oancea; Janet

#### Orchard, 2015

**Article** | **Recommended** | Trainees often see teacher training as having two discrete elements: campus and placement. This article discusses the imperative for critical reflection amongst teachers. It has not been shared with you for the purpose of citing within your assignment - it would be inappropriate to do so. What it does do, however, is highlight the importance of educational research as the third and necessary element linked to teachers' professional knowledge. It's well worth a read.

# Session 6: Multiplication (6 items)

**Good practice in primary mathematics: evidence from successful schools**, by OFSTED, 2011

Webpage | Recommended | While this document was withdrawn in 2018, its findings helped shape the 2014 National Curriculum for mathematics. Ofsted argues that less efficient approaches and algorithms involve too many steps (p.3), but this session considers the implications of teaching procedurally, which can be at the expense of children's deeper understanding.

Young Children's Intuitive Models of Multiplication and Division. - in Journal for Research in Mathematics Education, by Joanne T. Mulligan; Michael C Mitchelmore, 1997 Article | Recommended | This article makes clear that the term multiplicative relates to multiplication AND division, showing that these two concepts are interconnected. Our role, as teachers, is to expose these connections so that children can see the inverse relationship.

Multiplication: Lesson videos on elements of progression in multiplication across the primary school, by NCETM Webpage | Recommended

Six resources for teaching and learning times tables, by NCETM, 2020 Webpage | Recommended | Making connections when teaching the times tables

#### Number Facts, by NCETM

Webpage | Recommended | Look at the video clip for rapid recall in KS2 for multiplication facts - it gives a great activity but also models helping pupils to explain how they derived facts.

Understanding and teaching primary mathematics, by Tony Cotton, 2016 Book | Optional | Chapter 6 on calculation is essential reading. This is a very useful book for investigating other aspects of primary maths teaching. It will be useful to refer to for your assignment.

# Session 7: Division (3 items)

Division: what do we mean by 'efficient methods'? - in Mathematics Teaching, by Dave Benson, 2014 Article | Essential

Young Children's Intuitive Models of Multiplication and Division. - in Journal for Research in

Mathematics Education, by Joanne T. Mulligan; Michael C Mitchelmore, 1997 Article | Recommended

Multiplication and Division (Chapter 5) - in Primary mathematics for trainee teachers, by Ahir Balbir, 2014 Chapter | Recommended

# Session 8: Fractions, decimals and percentages (2 items)

**Fractions: Spine 3 of the Primary Mastery Professional Development Materials**, by NCETM **Webpage** | **Recommended** | In our session, our focus is very much on number lines and bar models. These representations are well used by the NCETM and make lovely connections to prior learning. Of particular use are the teacher guides, as they unpick the concepts really well.

Mathematics explained for primary teachers, by Derek Haylock, 2024 Book | Recommended | Refer to Haylock's sections on fractions, decimals and percentages (it doesn't matter which edition you are looking at) - a great resource to support you in making connections between concepts.

The next sections give a wider range of books, articles and websites which may be useful for assignment writing or for developing pedagogical understanding. Don't be overwhelmed - you don't have to read them all!

# Attainment grouping (5 items)

Educational triage and ability-grouping in primary mathematics: a case-study of the impacts on low-attaining pupils - in Research in Mathematics Education, by Rachel Marks, 2014\_\_\_\_

Article | Optional

Valid and valuable: lower attaining pupils' contributions to mixed attainment mathematics in primary schools - in Research in Mathematics Education, by Nancy Barclay, 2021 Article | Recommended | Barclay's award winning research focuses on the benefits if mixed ability pairings in maths. Whereas most research focuses on the benefits to children who struggle in maths, this research finds that it is those learners who will often notice what others do not. Fundamental to this is the role of the effective teacher.

Mastery mathematics: Changing teacher beliefs around in-class grouping and mindset - in Teaching and Teacher Education, by Pete Boyd; Andy Ash, 2018-10 Article | Optional

Attainment Grouping as self-fulfilling prophesy? A mixed methods exploration of self confidence and set level among Year 7 students - in International Journal of Educational Research, by Becky Francis; Paul Connelly; Louise Archer; Jeremy Hodgen; Anna Mazenod; David Pepper; Seaneen Sloan; Becky Taylor; Antonina Tereshchenko; Mary-Claire Travers, 2017

**Article** | **Optional** | While this does focus on year 7, the argument here is well linked to other researchers.

Numeracy for all learners: teaching mathematics to students with special needs, by Pamela D. Tabor; Dawn Dibley; Amy J. Hackenberg; Anderson Norton, 2021 Book | Recommended

### Representations in maths (8 items)

Tools for maths teachers, by MathsBot.com

Website | Recommended | This site, created by a maths teacher in England, is very useful and free to use. There's a large range of pictorial representations that can be used on interactive whiteboard.

**Magical hopes: manipulatives and the reform of math education** - in American Educator: The Professional Journal of the American Federation of Teachers, by Deborah Loewenberg Ball, 1992

**Article** | Ball discusses the misguided belief that manipulatives (and pictorial representations) are a cure all for maths, particularly when they are relied on exclusively to show/see the underlying structure. The author makes the argument that manipulatives, pictorials, context, discussion, reasoning, estimation and children's engagement all have a role to play.

Concrete, pictorial and abstra

Are we having fun yet? How teachers use manipulates to teach mathematics - in Educational Studies in Mathematics, by Patricia S. Moyer, 2001

**Article** | **Recommended** | This article discusses how teachers use manipulatives and potential issues. If you have use of manipulatives as a key decision for your assignment this article will be helpful to read and reference.

Using resources to support mathematical thinking: primary and early years, by Doreen Drews; Alice Hansen, 2007

**Book** | **Recommended** | If you are writing about using manipulatives as a key decision you may wish to reference this book.

Using a counting stick to teach the 17x table, by Jill Mansergh, 2009

**Audio-visual document** | Essential | Using a counting stick to learn connected facts, Jill Mansergh demonstrates using the 17x table.

The Use of a Bar Model Drawing to Teach Word Problem Solving to Students With Mathematics Difficulties - in Learning Disability Quarterly, by Lisa L. Morin; Silvana M. R. Watson; Peggy Hester; Sharon Raver, 2017-05

Article | Optional | This article may be useful to read if one of your key decisions is to use pictorial representations such as the bar model within your planning.

**GUEST POST: Manipulatives – Why They Can Hinder Learning and What You Can Do About It**, by Sara Fulmer, 2017

**Webpage** | **Recommended** | This blog post suggests that use of manipulatives needs to be carefully considered.

Using manipulatives in the foundations of arithmetic: Literature review, by Rose Griffiths; Sue Gifford; Jenni Back, 2017

**Document** | Recommended | This is a highly recommended research paper if you are

writing about using manipulatives in your assignment. It signposts major articles which you could use and should be aware of.

# Maths vocabulary (5 items)

Mathematical Vocabulary, by Department for Education and Employment, 2000
<b>Document</b>   Essential   This booklet was produced for the previous National Curriculum.
The content is still entirely relevant but some of the vocabulary may now be in different
year groups.

Mathematics glossary for teachers in Key Stages 1 to 3, by National Centre for Excellence in the Teaching of Mathematics, 2014 Document | Essential

Mathematical Vocabulary, by Department for Education and Employment, 2000 Document | Optional | This booklet was produced for the previous National Curriculum. The content is still entirely relevant but some of the vocabulary may now be in different year groups.

Maths vocabulary (NCETM Maths Podcast, Episode 71), by Julia Thomson; Victoria Moore, 2023-07-03

Audio document )| Optional

A Maths Dictionary for Kids: plus maths charts, by Jenny Eather Website ] Optional

#### Addressing misconceptions in maths (6 items)

There is no definitive guide on misconceptions but these provide a start. Don't forget the core texts at the top of the page.

Three practical approaches to help pupils learn from mathematical mistakes - in EEF Blog, by Simon Cox, 2020-09-25

Webpage | Recommended | This article is useful for thinking about planning in misconceptions.

Dealing with misconceptions in mathematics - in Issues in mathematics teaching, by Malcolm Swan, ©2001 Chapter | Chapter 10

Mathematical misconceptions: a guide for primary teachers, by Anne Cockburn; G. H. Littler, 2008 Book Recommended

Children's errors in mathematics, edited by Alice Hansen, 2020 Book Recommended

Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching, by Jo Boaler; Carol S. Dweck, 2015 Book Recommended | Chapter 2 is well worth reading in relation to misconceptions. It is entitled 'the power of the struggle'. This book is very readable and is highly recommended. The sections on mindset and fluency may also be useful for your assignment.

#### Topics in Depth Project, by Resourceaholic

Webpage | Optional | Go to the primary section of this page (scroll down). Although it's general content, it's broken down by year group and, within each strand of maths, you will find potential misconceptions.

#### Talk/ dialogic teaching in the mathematics classroom (9 items)

These resources should help you with your assignment - both with what to include in your unit plan and how to justify planning decisions made.

We need to talk (about maths), by National Centre for Excellence in the Teaching of Mathematics, 2025-02-05

Private talk, public conversation, by Mike Askew

**Document** | Recommended | This article is highly recommended if you are writing about the importance of promoting pupil talk in the maths classroom. Mike Askew is well respected.

**Talk** - in Transforming primary mathematics: understanding classroom tasks, tools, and talk, 2016

**Chapter** | **Recommended** | Chapter 11 builds on Askew's research, above.

Talking about maths - in Education 3-13, by Janet Evans, March 2002 Article | Recommended | Giving children something to talk about

Dialogic Teaching: Evaluation report and executive summary, by Tim Jay; Ben Willis; Peter Thomas; Roberta Taylor; Nick Moore; Cathy Burnett; Guy Merchant; Anna Stevens, 2017 Document ] Optional

Exploring talk in school, by Neil Mercer; Steve Hodgkinson, 2008 Book | Optional | Not specific to maths, but this book does talk about the importance of promoting pupil talk.

A dialogic teaching companion, by Robin J Alexander, 2020 Book | Optional

**5 Practices for Orchestrating Productive Mathematics Discussion**, by Margaret (Peg) Smith; Mary Kay Stein, 2018

**Book** | Optional | Smith and Stein identified 5 teacher actions to support talk in the maths classroom. Don't view the approach as 5 necessary steps but do consider what strategies you can adopt in relation to these and whether they will support children to think about their thinking.

Mathematical Discourse: Let the Kids Talk!, by Barbara Blanke, 2018 Book | Recommended