# PGCE primary mathematics (3-7 and 5-11) (Terms 1, 2 and 3)

This is the reading list for the maths element of the PGCE Core Subjects module. NB: Content will be added as the seminars progress across the year.

This PGCE module is designed to develop your conceptual understanding of maths, which will support their growing subject knowledge. Imperative to this is making connections across different strands of maths, understanding the ways in which children learn and typically develop, and the pedagogical approaches that can facilitate children's deeper understanding, irrespective of age. Focusing on some of the big ideas in maths, linked to the module on Professional Studies, trainees will see the way in which approaches can be seamlessly integrated across the maths curriculum.

This reading list covers the reading specific to each session, but has additional sections linked to topics that will be applicable when teaching.



#### 65 items

#### How to make the most of this reading list

The reading list has been divided into sections linked to a) your lectures/ seminars on campus, b) your professional studies module and c) considerations when teaching (e.g. CPA, ability grouping and maths vocabulary). Please do take advantage of all the links here (maths vocabulary, misconceptions etc.) and use them to support your wider reading.

#### Core documents (8 items)

These documents will support your teaching, in your trainee year and beyond. Please refer to these and develop your thinking around them.

#### **5-11** (2 items)

Mathematics explained for primary teachers, by Derek Haylock, 2024

Book | Essential | We strongly recommend that you buy this book, as it is only available as a hard copy and the library has limited quantities. It will be of particular use to you during placement (SBCs) and in your first few years of teaching, as it effectively plugs gaps in conceptual knowledge. Please bear in mind that older versions make very few/negligible references to mastery but, regardless of this, any of Haylock's books are designed to support deeper thinking, which we know is the essence of teaching for mastery.

National curriculum in England: primary curriculum, by Department for Education, 2015

Webpage | Essential | This document outlines the entitlement of any child (i.e. a statutory requirement) - it should become a guiding document for you whenever you're teaching, irrespective of whether your school follows a scheme or has its own curriculum.

Download this document in its entirety and use it, rather than specific maths versions, to support your teaching in years 1-6. Working wit the document in this way will increase your familiarity with all of it, and support you to make connections between maths and other subjects - the more we can link teaching, the more effective it will be. Even if you are teaching in nursery or EY, please do refer to the year 1 skills - it shows how learning progresses.

#### **3-7** (6 items)

**Small numbers, big ideas : essential concepts for teaching early maths**, by Jonathan Austen, 2024

Book | Recommended

**Mathematics in early years education**, by Ann Montague-Smith; Tony Cotton; Alice Hansen; Alison Price; Ann Montague-Smith, 2018

**Book** | **Essential** | This book is a great resource for any trainee or ECT teaching maths to younger children.

Early years foundation stage (EYFS) statutory framework for group and school-based providers, by Department for Education, Revised (effective 1st September, 2025)

Document | Essential | Mathematics features on page 10. While amounting to only a third of a page, it does demand that young children are supported to develop a deep understanding. Please be aware, however, that maths is everywhere and can be developed in very creative and inspirational ways (e.g through storybooks, loose parts play, outdoor experiences and even when using the equipment on the playground).

Development Matters: Non-statutory curriculum guidance for the early years foundation stage, by Department for Education, 2023

**Document** | Recommended | Non-statutory but relevant guidance for those working in EY

Birth to 5 Matters: non-statutory guidance for the Early Years Foundation Stage, by Early Years Coalition, 2021

Website | Recommended | This is non-statutory guidance for teachers and support staff working in nursery and EY. A lot of schools use this document for its guiding principles. It is full of great ideas to support teaching, and is a great resource when planning for maths - very practical, hands-on activities to develop children's skills.

#### Learning Trajectories

Website | Recommended | This website remains free of charge but does require you to sign up - no strings attached. You can access materials by area (e.g. counting) and there are plenty of activities to support teaching, but the real strength of this website is the learning trajectories (the typical stages of development when children are learning maths). These learning trajectories are diagnostic and will help you to recognise the preceding skills involved when teaching a concept which, when plugged, will enable children to progress to the next stage of learning. There are also very practical formative assessment materials available on the website (small group and 1-1 tools) that will support you to see what children have grasped.

## Introductory lecture: Subject knowledge and curriculum introduction (5 items)

This lecture references a lot of literature and documents pertinent to the session.

Improving Mathematics in the Early Years and Key Stage 1: Five recommendations to support practitioners in developing the maths skills of 3–7 year-olds, by Education Endowment Foundation, 2020

Webpage | Recommended | Guidance from the EEF (Education Endowment Foundation) about effective teaching of maths in EY and KS1. The guidance poster is particularly helpful to reference when planning lessons.

Improving Mathematics in Key Stages 2 and 3: Eight recommendations to improve outcomes in maths for 7-14 year olds, by Education Endowment Foundation, 2017

Webpage | Recommended | Guidance from the EEF (Education Endowment Foundation) about effective teaching of maths in KS2. The guidance poster is particularly helpful to reference when plannining lessons.

**Relational Understanding and Instrumental Understanding** - in Mathematics Teaching, by Richard R Skemp, 1976

Article | Recommended | On the 'Papers' section of Skemp's website is his article entitled 'Relational Understanding and Instrumental Understanding' - one of the most cited articles in maths education. The article focuses on the difference between teaching for understanding (relational) and teaching rules (instrumental). This will. hopefully, be one of those articles that forever resonates with you.

Computational Estimation Strategies of Professional Mathematicians - in Journal for Research in Mathematics Education, by Ann Dowker, 1992

Article | Optional

Why do Americans stink at math? Some of the answer - in Nonpartisan Education Review, by Wayne Bishop, 2014

Article | Optional

Fluency, reasoning and problem solving (applicable across all sessions) (5 items)

## Fluency (2 items)

Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts, by Jo Boaler, 2015

**Document** | Recommended | Make this the first article you read about fluency.

**Procedural Fluency in Mathematics**, by National Council of Teachers of Mathematics, 2014 **Webpage** | **Essential** | There is so much misunderstanding about procedural fluency but this page sets the record straight.

## Reasoning (2 items)

**Rich mathematical tasks** - in Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching, 2015

Chapter | Recommended

Reasoning: the Journey from Novice to Expert, by NRICH Primary Team, 2021

**Document** | Optional | Progression in reasoning

#### Problem solving (1 items)

**Teaching specific tactics for problem solving** - in Foster77 Mathematics Education: Colin Foster's mathematics education blog, by Colin Foster, 2023-03-30

Webpage | Recommended

#### Session 2: Number sense (5 items)

Subitising, counting and unitising

A proposed framework for examining basic number sense - in For the Learning of Mathematics, by Alistair McIntosh; Barbara J Reys; Robert E Reys, 1992

Document | Essential

Spatial Reasoning, by Early Childhood Maths Group

Webpage | Optional | This is a UK website, developed by a leading group of EY mathematics practitioners and researchers.

**Subitizing: What Is It? Why Teach It?** - in Teaching children mathematics, by Douglas H Clements, 1999

Article | Recommended

The counting model - in The child's understanding of number, ©1986

**Chapter** | **Recommended** | Gelman and Gallistel's work around the counting principles are clearly explained in Chapter 7.

What is unitising, and why is it important?, by National Centre for Excellence in the Teaching of Mathematics, 2019-11-06

Webpage | Recommended | Some very brief and helpful guidance around unitising for all year groups

#### Session 3: Place value (1 items)

Place value: the English disease? - in Enhancing primary mathematics teaching, 2003 Chapter | Optional

#### Session 4: Addition and subtraction (2 items)

The small steps in learning - coherence

**Progression Maps for Key Stages 1 and 2**, by National Centre for Excellence in the Teaching of Mathematics

Webpage | Recommended | Helpful in seeing the way learning develops

Young Children's Intuitive Models of Multiplication and Division. - in Journal for Research in Mathematics Education, by Joanne T. Mulligan; Michael C Mitchelmore, 1997

Article | Recommended | This article makes clear that the term multiplicative relates to multiplication AND division, showing that these two concepts are interconnected. Our role, as teachers, is to expose these connections so that children can see the inverse relationship.

#### Session 5: Multiplication and division (5 items)

Formative assessment

**Division: what do we mean by 'efficient methods'?** - in Mathematics Teaching, by Dave Benson, 2014

Article | Essential

Good practice in primary mathematics: evidence from successful schools, by OFSTED Webpage | Optional | While this document was withdrawn in 2018, its findings helped shape the 2014 National Curriculum for mathematics. Ofsted argues that less efficient approaches and algorithms involve too many steps (p.3), but this session considers the implications of teaching procedurally, which can be at the expense of children's deeper understanding.

#### Primary Assessment Materials, by NCETM, 2015

Webpage | Recommended | This site provides a useful suite of activities that were designed to assess children's understanding. You can locate materials by year group, and within each year group you will see the range of activities provided. Once you have taught a concept, the tasks provided in this book can provide a useful way to evidence children's understanding.

Strategies for Formative Assessment, by National Council of Teachers of Mathematics Webpage | Recommended | A precis of strategies to support formative assessment

Using Formative Assessment Effectively, by Trena Wilkerson, 2022

**Webpage** | **Optional** | A specific focus on classroom based formative assessment as a means to identifying what children have understood in a lesson and how it informs next steps

#### Session 6: Dimension (4 items)

Variation theory

**Understanding measurement** - in Understanding mathematics for young children: a guide for teachers of children 3-7, 2017

Chapter | Recommended | Chapter 8 gives a detailed account of the range of things that primary children need to measure and some key ideas (such as conservation and the need for direct and indirect comparison). You are advised to buy a copy of this book as it is useful throughout the course. No e-book is available as the publisher refuses to sell the e-book version to libraries; the View Online link here is to this chapter only.

**Shape and space** - in Mathematics in early years education, edited by Ann Montague-Smith , 2018

**Chapter** | Optional | Chapter 6 will support your development of associated pedagogy

#### around teaching geometry

**Variation theory** - in Transforming primary mathematics: understanding classroom tasks, tools, and talk, 2016

**Chapter** | **Recommended** | Mike Askew's work is a straightforward and sensible read, supporting you to better undersanding variation theory and what it entails.

Teaching with Procedural Variation: A Chinese Way of Promoting Deep Understanding of Mathematics - in International Journal for Mathematics Teaching and Learning, by Mun Yee Lai; Sarah Murray, 2012-04-19

Article | Optional

#### Session 7: Equivalence (5 items)

**Relational Understanding and Instrumental Understanding** - in Mathematics Teaching, by Richard R Skemp, 1976

Article | Essential

Coordinating invisible and visible sameness within equivalence transformations of numerical equalities by 10- to 12-year-olds in their movement from computational to structural approaches - in ZDM - Mathematics Education, by Carolyn Kieran; Cesar Martínez-Hernández, 2022

Article | Essential

**Concepts associated with the equality symbol** - in Educational Studies in Mathematics, by Carolyn Kieran, 1981

Article | Optional

**Myths of Early Math** - in Education Sciences, by Douglas Clements; Julie Sarama, 2018-05-17

Article | Recommended

Keys to the gate? equal sign knowledge at second grade predicts fourth-grade algebra competence - in Child development, by Percival G. Matthews, 2020 Article | Optional

### Session 8: Self regulation and maths (3 items)

What Teachers Think about Self-Regulated Learning: Investigating Teacher Beliefs and Teacher Behavior of Enhancing Students' Self-Regulation - in Education Research International, by Charlotte Dignath-van Ewijk; Greetje van der Werf, 2012

Article | Recommended

Creative teaching: mathematics in the primary classroom, by Mary Briggs; Sue Davis, 2015

Book | Optional

Bringing math home : a parent's guide to elementary school math: games + activities + projects, by Suzanne L. Churchman, 2006

Book | Optional

#### Attainment grouping (2 items)

Valid and valuable: lower attaining pupils' contributions to mixed attainment mathematics in primary schools - in Research in Mathematics Education, by Nancy Barclay, 2021

Article | Barclay's award winning research focuses on the benefits of mixed ability pairings in maths. Whereas most research focuses on the benefits to children who struggle in maths, this research finds that it is those learners who will often notice what others do not. Fundamental to this is the role of the effective teacher.

Attainment Grouping as self-fulfilling prophesy? A mixed methods exploration of self confidence and set level among Year 7 students - in International Journal of Educational Research, by Becky Francis; Paul Connelly; Louise Archer; Jeremy Hodgen; Anna Mazenod; David Pepper; Seaneen Sloan; Becky Taylor; Antonina Tereshchenko; Mary-Claire Travers, 2017

Article | While this does focus on year 7, the argument here is well linked to other researchers.

#### Concrete, pictorial and abstract (7 items)

Although it's an initialism, CPA is one of the current buzzwords in maths, but there's still a lot of misunderstanding around it. If you want to be informed, we strongly recommend that you read these articles.

**Magical hopes: manipulatives and the reform of math education** - in American Educator: The Professional Journal of the American Federation of Teachers, by Deborah Loewenberg Ball, 1992

Article | Optional | Ball discusses the misguided belief that manipulatives (and pictorial representations) are a cure all for maths, particularly when they are relied on exclusively to show/see the underlying structure. The author makes the argument that manipulatives, pictorials, context, discussion, reasoning, estimation and children's engagement all have a role to play.

Are We Having Fun Yet? How Teachers Use Manipulatives to Teach Mathematics - in Educational Studies in Mathematics : An International Journal, by Patricia S. Moyer, 2001 Article | Optional | Moyer discusses the notion that teachers often see the maths manipulatives (concrete) as a way to have fun in maths. Instead, the author talks about the importance of how these resources should be used, often in support of very specific teaching.

Using resources to support mathematical thinking: primary and early years, by Doreen Drews; Alice Hansen, 2007

Book | Optional

Using a counting stick to teach the 17x table, by Jill Mansergh, 2009

Audio-visual document
| See how a counting stick can be used to support your teaching of the times tables.

The role of virtual manipulatives on the CPA approach - in The Electronic Journal of Mathematics and Technology, by Ngan Hoe Lee; Boon Leong Jeremy Tan

Article

Tools for maths teachers, by MathsBot.com

**Website** | This website offers virtual manipulatives (aka pictorials) that are dynamic and can be used successfully on an interactive whiteboard.

Enhancing students' mathematical problem-solving skills through bar model visualisation technique - in International Electronic Journal of Mathematics Education, by Sharifah Osman; Che Nurul Azieana Che Yang; Mohd Salleh Abu; Norulhuda Ismai; Hanifah Jambari; Jeya Amantha Kumar

Article | Useful to see the way in which bar models can support problem solving

#### Maths talk (3 items)

Private talk, public conversation, by Mike Askew Document

**Talk** - in Transforming primary mathematics: understanding classroom tasks, tools, and talk, 2016

Chapter | Chapter 11 builds on Askew's research, above.

Talking about maths - in Education 3-13, by Janet Evans, March 2002

Article | Giving children something to talk about

#### Maths vocabulary (5 items)

Mathematics glossary for teachers in Key Stages 1 to 3, by National Centre for Excellence in the Teaching of Mathematics, 2014

**Document** | **Optional** | This is a very helpful resource, particularly when you're introducing a new concept.

Maths vocabulary (NCETM Maths Podcast, Episode 71), by Julia Thomson; Victoria Moore, 2023-07-03

Audio document | Optional

Effective questioning and responding in the mathematics classroom, by John Mason, 2010 Document | Optional

A Maths Dictionary for Kids: plus maths charts, by Jenny Eather Website

Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching, by Jo Boaler; Carol S. Dweck, 2015

Book

## Misconceptions in maths (5 items)

Mathematical misconceptions: a guide for primary teachers, by Anne Cockburn; G. H. Littler, 2008

Book | Recommended

Dealing with misconceptions in mathematics - in Issues in mathematics teaching, by

Malcolm Swan, ©2001

Chapter | Optional

The power of mistakes and struggle - in Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching, 2015

Chapter | Optional | Chapter 2 is well worth reading in relation to misconceptions.

Three practical approaches to help pupils learn from mathematical mistakes - in EEF Blog, by Simon Cox, 2020-09-25

Webpage | Optional | Note for students

This article is useful for thinking about planning in misconceptions

Topics in Depth Project, by Resourceaholic

**Webpage** | **Recommended** | Go to the primary section of this page (scroll down). Although it's general content, it's broken down by year group and, within each strand of maths, you will find potential misconceptions.